1 In Fig. 6, OAB is a thin bent rod, with OA = a metres, AB = b metres and angle OAB = $120°$. The bent rod lies in a vertical plane. OA makes an angle θ above the horizontal. The vertical height BD of B above O is h metres. The horizontal through A meets BD at C and the vertical through A meets OD at E.

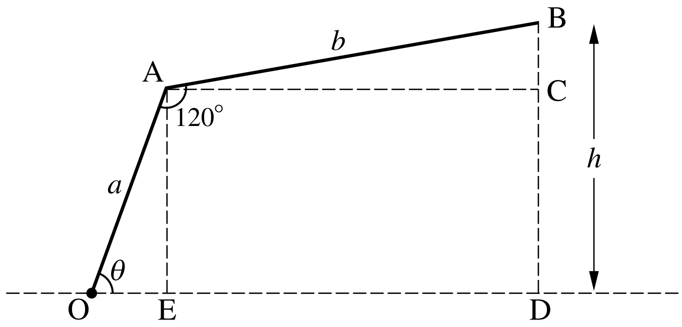

Fig. 6

(i) Find angle BAC in terms of θ. Hence show that

$$h = a\sin\theta + b\sin(\theta - 60°).$$ **[3]**

(ii) Hence show that $h = (a + \frac{1}{2}b)\sin\theta - \frac{\sqrt{3}}{2}b\cos\theta$. **[3]**

The rod now rotates about O, so that θ varies. You may assume that the formulae for h in parts **(i)** and **(ii)** remain valid.

(iii) Show that OB is horizontal when $\tan\theta = \dfrac{\sqrt{3}b}{2a+b}$. **[3]**

In the case when $a = 1$ and $b = 2$, $h = 2\sin\theta - \sqrt{3}\cos\theta$.

(iv) Express $2\sin\theta - \sqrt{3}\cos\theta$ in the form $R\sin(\theta - \alpha)$. Hence, for this case, write down the maximum value of h and the corresponding value of θ. **[7]**

2 Express $3\cos\theta + 4\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

Hence solve the equation $3\cos\theta + 4\sin\theta = 2$ for $-\pi \leqslant \theta \leqslant \pi$. [7]

3 Show that the equation $\operatorname{cosec} x + 5\cot x = 3\sin x$ may be rearranged as

$$3\cos^2 x + 5\cos x - 2 = 0.$$

Hence solve the equation for $0° \leqslant x \leqslant 360°$, giving your answers to 1 decimal place. **[7]**

4 Part of the track of a roller-coaster is modelled by a curve with the parametric equations

$$x = 2\theta - \sin\theta, \quad y = 4\cos\theta \quad \text{for } 0 \leqslant \theta \leqslant 2\pi.$$

This is shown in Fig. 8. B is a minimum point, and BC is vertical.

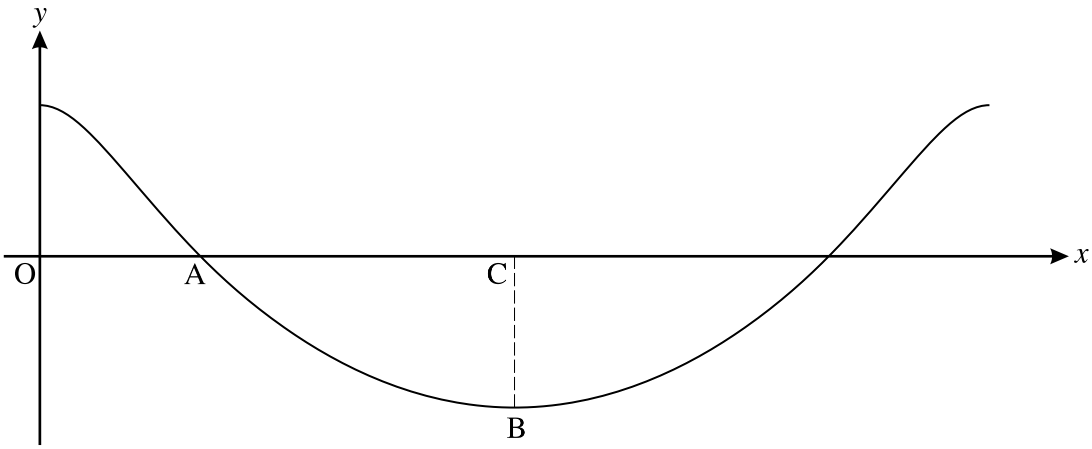

Fig. 8

(i) Find the values of the parameter at A and B.

Hence show that the ratio of the lengths OA and AC is $(\pi - 1) : (\pi + 1)$. **[5]**

(ii) Find $\dfrac{dy}{dx}$ in terms of θ. Find the gradient of the track at A. **[4]**

(iii) Show that, when the gradient of the track is 1, θ satisfies the equation

$$\cos\theta - 4\sin\theta = 2.$$ **[2]**

(iv) Express $\cos\theta - 4\sin\theta$ in the form $R\cos(\theta + \alpha)$.

Hence solve the equation $\cos\theta - 4\sin\theta = 2$ for $0 \leqslant \theta \leqslant 2\pi$. **[7]**

5 Express $4\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

Hence solve the equation $4\cos\theta - \sin\theta = 3$, for $0 \leqslant \theta \leqslant 2\pi$. **[7]**